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FLOW OF A STREAM OF UNEVENLY HEATED LIQUID OVER A GAS BUBBLE AT LOW MARANGONI NUMBERS

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The problem of the thermocapillary convection in an unevenly heated liquid near a gas bubble is solved analytically. Estimates are given for the velocity of drift and the shape of the bubble and the vortex boundary.

Let a gas bubble of radius a be placed in a liquid which fills the entire space. A constant temperature gradient $\nabla T = \mathbf{A}$ is maintained at infinity. The force of gravity is absent. Shear stresses producing thermocapillary convection in the liquid develop under these conditions owing to the temperature dependence of the coefficient of surface tension α at the surface of the bubble. The bubble itself begins to move. Under steady conditions the velocity u of this translational motion is constant and is determined in the course of the solution.

The problem can be formulated as a steady-state problem if one changes to a frame of reference connected with the bubble. In such a system the velocity of oncoming flow of the liquid is equal to the drift velocity of the bubble with the opposite sign.

In the report it is assumed that the gas in the bubble is thermally nonconducting and its viscosity is vanishingly small. This allows us not to write the Navier–Stokes equation and the heat-conduction equation for the gas. However, the pressure q in the bubble must be taken into account in writing the boundary conditions.

The problem will be solved in dimensionless quantities. For this we take the following as the characteristic dimensions: the radius a of the undisturbed bubble for the length $|d\alpha/dT|Aa/\eta$ for the velocity, Aa for the temperature, and $|d\alpha/dT|A$ for the pressure. Then the steady distributions of velocities $\mathbf{v}$, pressures p, and temperatures T in the liquid are determined by the system of equations

$$M(\mathbf{v}\nabla)\mathbf{v} = -\nabla p + \Delta\mathbf{v};\ \nabla\mathbf{v} = 0;\ MP(u + \mathbf{v}\nabla T) = \Delta T. \tag{1}$$

Here all the quantities are dimensionless; $M = |d\alpha/dT|(A\alpha^2/\nu\eta)$ and $P = \nu/\chi$ are the Marangoni and Prandtl numbers. The appearance of the drift velocity u in the heat-conduction equation is connected with the choice of the reference point of the temperature. One can assume that the motion is already established by the starting time. Then it is convenient to measure the temperature from the undisturbed temperature of that point of space at which the bubble is found at the time under consideration upon its continued uniform motion. The partial derivative with respect to time in the nonsteady equation of heat conduction also gives a term proportional to u. The corresponding term of the Navier–Stokes equation vanishes in the chosen frame of reference.

The boundary conditions at the surface of the bubble must be added to the system (1). We take the free surface of the bubble as impermeable and thermally nonconducting, and therefore the normal components of the velocity and heat flux and the normal and tangential components of the stresses vanish at the surface. We write these conditions in a spherical coordinate system r, θ, φ with the polar axis parallel to the vector $\mathbf{A}$.

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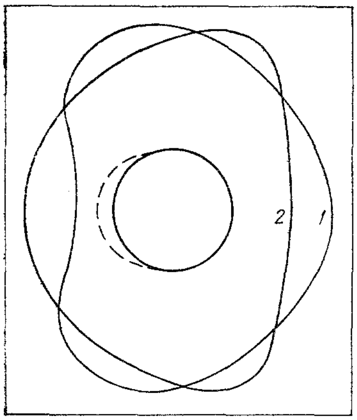

Fig. 1. Meridional cross sections of bubbles in hypothetical liquids of the water type (curve 1) and the mercury type (curve 2). The boundary of the steady vortex (dashes) behind an air bubble in water (solid line) with M = 0.715 is shown at the center of the figure at a smaller scale.

We will assume that the bubble is a body of rotation with the axis of symmetry along the polar axis. The surface of the bubble is given by the equation $r = R(\theta)$. The function $R(\theta)$ must be found in the course of the solution of the problem. We take the coefficient of surface tension as decreasing linearly with temperature. After transformations [1] we obtain

$$v_r = \frac{R'}{R} v_\theta; \quad \frac{\partial T}{\partial r} = \frac{R'}{R^2}\frac{\partial T}{\partial \theta};$$

$$p - q + \left(\frac{\alpha_0}{M} - T\right) 2H = -2\frac{\partial v_r}{\partial r} - \frac{R'}{R}\left[\sigma_{r\theta} - \frac{R'\frac{\partial T}{\partial r} + \frac{\partial T}{\partial \theta}}{(R^2 + R'^2)^{1/2}}\right];$$

$$\frac{R'}{R}\left[p - q + \left(\frac{\alpha_0}{M} - T\right) 2H\right] = \frac{R'\frac{\partial T}{\partial r} + \frac{\partial T}{\partial \theta}}{(R^2 + R'^2)^{1/2}} - \sigma_{r\theta}\frac{2R'}{R^2}\left(\frac{\partial v_\theta}{\partial \theta} + v_r\right). \tag{2}$$

Here $\alpha_0 = \alpha a/\eta\nu$ is the dimensionless coefficient of surface tension; $2H = \frac{2R^2 + 3R'^2 - RR''}{(R^2 + R'^2)^{3/2}} - \frac{\cot\theta R'}{(R^2 + R'^2)^{1/2} R}$ is the average curvature of the surface [2]; $\sigma_{r\theta} = \frac{1}{R}\frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{R}$.

A uniform flow, a constant temperature gradient, and zero pressure are assigned at infinity:

$$v_r = -u\cos\theta, \quad v_\theta = u\sin\theta, \quad T = r\cos\theta, \quad p = 0, \tag{3}$$

We will solve the boundary-value problem (1)-(3) by the method of joined asymptotic expansions [3, 4]. Following this method, we divide the entire space into an outer $0(M^{-1}) < r \le \infty$ and an inner $1 \le r \le 0(M^{-1})$ region. The expansions v_*, p_*, and T_* in the inner region (the Stokes expansions) are valid for $M \to 0$ and a fixed coordinate r and satisfy Eqs. (1) and (2). The boundary conditions (3) do not apply for these expansions. The expansions v^*, p^*, and T^* in the outer region (the Oseen expansions) are written in the variables $\rho = Mr$, $T^* = MT_*$, $v^* = v_*$, and $p_* = Mp^*$ and satisfy the boundary conditions (3) and the equations

$$(\mathbf{v}^*\nabla)\,\mathbf{v}^* = -\nabla p^* + \Delta \mathbf{v}^*; \quad \nabla \mathbf{v}^* = 0; \quad P(u + \mathbf{v}^*\nabla T^*) = \Delta T^*. \tag{4}$$

The solutions of the stated problems in the inner and outer regions are the following expressions:

$$\mathbf{v}_* = \mathbf{v}_{*0} + M\mathbf{v}_{*1} + M^2\mathbf{v}_{*2};\ p_* = Mp_{*1} + M^2p_{*2};\tag{5}$$

$$T_* = T_{*0} + MT_{*1} + M^2T_{*2};\ R(\theta) = 1 + M^2s_2P_2 + M^3s_3P_3;$$

$$\mathbf{v}^* = \mathbf{v}_0^* + M^3v_3^*;\ T^* = T_0^* + M^3T_3^*;$$

$$p^* = M^3p_3^*;\ q_0 = \frac{2\alpha_0}{M} + Mq_1;\ u = \frac{1}{2} + M^2u_2;$$

$$\mathbf{v}_{*0} = \frac{1}{2}\left(\frac{1}{r^3} - 1\right)P_1\mathbf{r}_1 + \frac{1}{2}\left(-\frac{1}{2r^3} - 1\right)r\nabla P_1;$$

$$T_{*0} = \left(r + \frac{1}{2r^2}\right)P_1;\ q_1 = -\frac{1+2P}{16};\ \mathbf{v}_{*1} = \frac{9}{64}\left(\frac{1}{r^4}\right.$$

$$\left.-\frac{1}{r^2}\right)P_2\mathbf{r}_1 - \frac{3\nabla P_2}{64r^3};$$

$$T_{*1} = P\left[\left(\frac{1}{12r} - \frac{1}{48r^4}\right) + \left(\frac{1}{9r^3} - \frac{1}{6r} - \frac{1}{24r^4}\right)P_2\right];$$

$$p_{*1} = -\frac{1}{16r^6} - \left(\frac{1}{32r^3} + \frac{1}{16r^6}\right)P_2;\ s_2 = -\frac{1}{\alpha_0}\left(\frac{15}{128} + \frac{7P}{144}\right);$$

$$\mathbf{v}_{*2} = \left[\left(-\frac{9}{1280r^4} - \frac{9}{640r} + \frac{d_1}{r^3} - u_2\right)P_1\mathbf{r}_1 + \left(\frac{9}{1280r^4}\right.\right.$$

$$\left.\left.-\frac{9}{1280r} - \frac{d_1}{2r^3} - u_2\right)r\nabla P_1\right] + \left[\left(\frac{9}{640r} + \frac{63}{2560r^4} + \frac{b_3}{r^3}\right.\right.$$

$$\left.\left.+\frac{d_3}{r^5}\right)P_3\mathbf{r}_1 + \left(\frac{9}{7680r} - \frac{21}{5120r^4} - \frac{b_3}{12r^3} - \frac{d_3}{4r^5}\right)r\nabla P_3\right];$$

$$p_{*2} = \frac{27}{640}\left(\frac{1}{r^5} - \frac{1}{r^7}\right)P_1 + \left(\frac{9}{128r^4} + \frac{81}{1280r^5}\right.$$

$$\left.-\frac{9}{320r^7} + \frac{5b_3}{2r^4}\right)P_3;$$

$$T_{*2} = \left(A_0 + \frac{A_2}{r^2} + \frac{A_3}{r^3} + \frac{A_5}{r^5} + \frac{A_6}{r^6}\right)P_1 + \left(B_0 + \frac{B_2}{r^2} + \frac{B_3}{r^3}\right.$$

$$\left.+\frac{B_4}{r^4} + \frac{B_5}{r^5} + \frac{B_6}{r^6}\right)P_3;$$

$$A_0 = \frac{9}{160}\left(\frac{9}{2} - \frac{26P}{3}\right);\ A_2 = -\frac{3s_2}{10} + \frac{P}{160}\left(\frac{11P}{9} - \frac{3}{2}\right);$$

$$A_3 = -\frac{P}{320}\left(\frac{2P}{3} + \frac{9}{2}\right);\ A_5 = -\frac{P}{1440}\left(8P + \frac{27}{4}\right);\ A_6 = \frac{P^2}{320};$$

$$B_0 = \frac{P}{960}\left(12P + \frac{27}{4}\right);\ B_2 = -\frac{P}{800}\left(\frac{40P}{3} + \frac{45}{4}\right);$$

$$B_3 = \frac{P}{480}\left(6P - \frac{27}{4}\right);\ B_4 = \frac{9s_2}{10} + \frac{P}{160}\left(P + \frac{135}{32}\right);\ B_5$$

$$= -\frac{P}{640}\left(\frac{16P}{3} + \frac{9}{2}\right);\ B_6 = \frac{P^2}{480};\ b_3 = \frac{27s_2}{35}$$

$$-\frac{1}{160}\left(\frac{153}{28} - \frac{27P}{112} + \frac{8P^2}{7}\right);$$

$$d_3 = \frac{36s_2}{35} - \frac{1}{160}\left(\frac{81}{112} + \frac{27P}{112} - \frac{8P^2}{7}\right);\ d_1 = \frac{s_2}{2}$$

$$+\frac{1}{320}\left(\frac{9}{2} + 3P - \frac{49P^2}{9}\right);$$

$$u_2 = \frac{4s_2}{5} + \frac{1}{320}\left(3P - \frac{49P^2}{9} - \frac{9}{4}\right);\ s_3 = \frac{1}{10\alpha_0}\left(\frac{P^2}{168}\right.$$

$$-\frac{9P}{7168}-\frac{153}{1792}-\frac{9s_2}{14}\Big); \quad \mathbf{v}_0^* = -\frac{1}{2}P_1\mathbf{r}_1-\frac{1}{2}r\nabla P_1;$$

$$T_0^* = \rho\cos\theta; \quad \mathbf{v}_3^* = \Big\{-\frac{P_1}{16\rho^3}+\frac{3}{512}\Big[\Big(\frac{2}{\rho}+\frac{8}{\rho^2}\Big) + \Big(\frac{24}{\rho^2}+\frac{96}{\rho^3}\Big)P_1-\Big(\frac{2}{\rho}+\frac{8}{\rho^2}\Big)P_2\Big]\exp\Big[-\frac{\rho(1+\cos\theta)}{4}\Big]\Big\}\mathbf{r}_1$$
$$-\Big\{\frac{P_1'}{32\rho^3}+\frac{3}{512}\Big[\Big(\frac{3}{\rho}+\frac{12}{\rho^2}+\frac{48}{\rho^3}\Big)P_1'+\Big(\frac{1}{\rho}+\frac{4}{\rho^2}\Big)P_2'\Big] \times \exp\Big[-\frac{\rho(1+\cos\theta)}{4}\Big]\Big\}\vec{\theta}_1; \quad P_3^* = -\frac{P_2}{32\rho^3};$$

$$T_3^* = \frac{P_1}{16\rho^2}+\Big[\frac{P(32P-59)}{192(P-1)\rho}+\frac{P(16P-7)}{128(P-1)}\Big(\frac{2}{\rho}+\frac{8}{P\rho^2}\Big)P_1\Big] \times \exp\Big[-\frac{\rho P(1+\cos\theta)}{4}\Big]+\Big[\frac{2}{\rho}-\Big(\frac{2}{\rho}+\frac{8}{\rho^2}\Big)P_1\Big]\frac{9P}{128(P-1)} \times \exp\Big[-\frac{\rho(1+\cos\theta)}{4}\Big]+\frac{P(32P-27)}{768}.$$

In the problem under consideration there is no term proportional to r^{-1} (the "Stokselet") in the expressions for the velocity, owing to which the second approximation in the Stokes problem is absent. Therefore, the expansion for the velocity proves to be uniformly applicable in the entire space up to the third term.

The equations written above were constructed so as to reduce to zero the mass flux through any surface including the bubble. These are the only possible expressions. Any other functions will have singularities of higher order at the origin and consequently will prove not to be joinable with (5). Using Eqs. (5) one can construct a uniformly applicable composite expansion. With its help we construct the stream function ψ:

$$\psi = -\sin^2\theta\,(r-1+M^2s_2P_2)\Big\{\frac{1}{4}(r^4+r^3+r^2)+\frac{9(r^2+r)}{128}\cos\theta + \frac{M^2}{2}\Big[\Big(\frac{4s_2}{5}-\frac{9}{1280}+\frac{3P}{320}-\frac{49P^2}{2880}\Big)r^4+\Big(\frac{7s_2}{10} + \frac{9}{1280}+\frac{3P}{320}-\frac{49P^2}{2880}\Big)r^3+\Big(\frac{3s_2}{5}+\frac{9}{1280}+\frac{3P}{320} -\frac{49P^2}{2880}\Big)r^2-\frac{9r}{1280}\Big]-\frac{M^2(15\cos^2\theta-3)}{24}\Big[\Big(\frac{3s_2}{5}-\frac{9}{640}\Big)r^3 + \Big(\frac{6s_2}{5}-\frac{9}{640}\Big)r^2+\Big(\frac{36s_2}{35}+\frac{360-27P+128P^2}{17920}\Big)r + \Big(\frac{36s_2}{35}-\frac{81+27P-128P^2}{17920}\Big)\Big]\Big\}. \tag{6}$$

This expression is reduced to zero not only along the axis of symmetry $\theta = 0$ and at the bubble surface $r = 1 + M^2s_2P_2$, but also on the curve whose equation is obtained if one equates the curly brackets in (6) to zero. This equation approximately describes the boundary of the steady vortex behind the bubble. For an air bubble in water ($P = 7.1$, $\alpha_0 = 73{,}000$) a vortex appears at $M_* = 0.715$. The shape for $M = 0.715$ is shown in Fig. 1 by a dashed line. The deformation of the bubble (solid line) for the air–water system is negligibly small. Therefore, the shapes of bubbles in two different hypothetical liquids of the water type (curve 1: large P and large α_0) and the mercury type (curve 2: small P but large α_0) are shown in the figure at a larger scale. It is seen that the shape of a bubble in a liquid with a large P is qualitatively similar to the shape of a bubble moving in water under the effect of the force of gravity. With small P the forward part of the bubble is flattened while the after part is drawn out.

We note that since the bubble moves uniformly the total force of resistance is equal to zero: The stresses produced by surface forces are compensated by viscous forces of friction.

NOTATION

T, temperature; v, velocity; p, pressure; ν, coefficient of kinematic viscosity; η, coefficient of dynamic viscosity; χ, thermal diffusivity; $\varkappa$, thermal conductivity; r, θ, φ, coordinates; a, bubble radius; A, constant temperature gradient; u, drift velocity; 2H, average curvature of surface; M, M$*$, P, α_0, dimensionless parameters of problem; differentiation with respect to the coordinate θ is denoted by a prime; P_l, Legendre polynomials of order l; α, coefficient of surface tension; r_1, $\vec{\theta}_1$, unit vectors of spherical coordinate system.

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VISCOSITY OF A WATER-FLUIDIZED BED

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The method of damping of oscillations of a ball submerged in a fluidized bed is used to study the viscosity of the bed.

The rheological properties of an air-fluidized bed have been studied rather extensively and by different methods. There are experimental data which we obtained [1, 2] allowing one to draw certain conclusions concerning the dependence of the effective viscosity of the bed on the properties of the solid phase.

To clarify the mechanism of the effective viscosity and the laws of its variation it is necessary to study beds fluidized by different agents, and therefore it is advisable to make measurements in a bed fluidized by water. These measurements present definite difficulties, since in its rheological properties a strongly rarefied bed approaches the properties of the fluidizing agent, the viscosity of which is low. Using the method which we developed [3], which provides for the motion of bodies in the bed in the region of small Reynolds

TABLE 1. Characteristics of Substances Used for Calibration

Substance, % at t, °C	$\rho\cdot10^{-3}$, kg/m^3	$\mu\cdot10$, N·sec/m^2	N_{exp}	$\nu\cdot10^4$, m^2/sec	$N_{exp}\,\rho\cdot10^{-3}$, kg/m^3
Water at 20	1,0	$1{,}005\cdot10^{-2}$	55,5	0,01	55,5
Aqueous solution of sugar					
20 at 21	1,08	$1{,}96\cdot10^{-2}$	40	0,018	43,20
40 at 20	1,18	$6{,}2\cdot10^{-2}$	25,8	0,053	30,44
60 at 34	1,29	$27{,}97\cdot10^{-2}$	14	0,217	18,06
60 at 30	1,29	$33{,}78\cdot10^{-2}$	13,2	0,262	17,03
60 at 25	1,29	$43{,}86\cdot10^{-2}$	11,0	0,340	14,19
60 at 20	1,29	$56{,}5\cdot10^{-2}$	9,5	0,438	12,25
Glycerin	1,24	3,68	4	2,968	4,96
Castor oil	0,95	9,03	2	9,505	1,90

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